Corrigés — Bases de la trigonométrie

Chapitre 9

Conversion degré ↔ radian

Solution 1.

Multiplier par $\frac{\pi}{180}$:

- $30° = \frac{\pi}{6}$
- $75° = \frac{75\pi}{180} = \frac{5\pi}{12}$
- $150° = \frac{5\pi}{6}$
- $210° = \frac{7\pi}{6}$
- $-135° = -\frac{3\pi}{4}$

Solution 2.

Multiplier par $\frac{180}{\pi}$:

- $\frac{\pi}{6} = 30°$;
- $\frac{3\pi}{4} = 135°$;
- $\frac{5\pi}{3} = 300°$;
- $-\frac{\pi}{2} = -90°$;
- $\frac{7\pi}{6} = 210°$.

cos, sin, tan

Solution 3.

- $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$
- $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
- $\tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$
- $\cos(\pi) = -1$
- $\sin\left(\frac{3\pi}{2}\right) = -1$
- $\tan(0) = 0$

Solution 4.

$\sin^2 x = 1 - \cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$, donc $\sin x = \pm\frac{4}{5}$. Comme $x \in [-\frac{\pi}{2}, 0]$, on est dans le 4ᵉ quadrant : $\sin x < 0$. Donc $\sin x = -\frac{4}{5}$.

$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{4}{5}}{\frac{3}{5}} = -\frac{4}{3}$.

Solution 5.
$\cos^2 x = 1 - \sin^2 x = 1 - \frac{4}{9} = \frac{5}{9}$, donc $\cos x = \pm\frac{\sqrt{5}}{3}$. Sur $\left[\pi, \frac{3\pi}{2}\right]$, $\cos x < 0$, donc $\cos x = -\frac{\sqrt{5}}{3}$.

$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{2}{3}}{-\frac{\sqrt{5}}{3}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$.

Identité fondamentale

Solution 6.
(a) $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = \frac{\sin^2 x + (1+\cos x)^2}{\sin x(1+\cos x)}$.

Le numérateur : $\sin^2 x + 1 + 2\cos x + \cos^2 x = 1 + 1 + 2\cos x = 2(1+\cos x)$.

Donc l'expression vaut $2\frac{1+\cos x}{\sin x(1+\cos x)} = \frac{2}{\sin}x$.

(b) $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \cos(2x) \times 1 = \cos(2x)$.

Solution 7.
$A = \sin^2 x + \cos^2 x + 3\tan^2 x = 1 + 3\tan^2 x$.

Formules de réduction

Solution 8.
- $\cos\left(\frac{5\pi}{6}\right) = \cos\left(\pi - \frac{\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$.
- $\sin\left(\frac{7\pi}{4}\right) = \sin\left(2\pi - \frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.
- $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$.
- $\cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$.
- $\sin\left(\frac{11\pi}{6}\right) = \sin\left(2\pi - \frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$.

Solution 9.
(a) $\cos(\pi - x) = -\cos x$; $\cos(\pi + x) = -\cos x$; $\cos(-x) = \cos x$. Donc $A = -\cos x - \cos x - \cos x = -3\cos x$.

(b) $\sin\left(\frac{\pi}{2} + x\right) = \cos x$; $\sin\left(\frac{\pi}{2} - x\right) = \cos x$; $\sin(\pi - x) = \sin x$. Donc $B = \cos x - \cos x + \sin x = \sin x$.

Solution 10.
1. $\cos x = \frac{\sqrt{3}}{2}$: $\alpha = \frac{\pi}{6}$. Sur $[0, 2\pi]$: $S = \left\{\frac{\pi}{6}, \frac{11\pi}{6}\right\}$.
2. $\sin x = -\frac{1}{2}$: $\alpha = -\frac{\pi}{6}$ donc sur $[0, 2\pi]$: $S = \left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$.
3. $\tan x = 1$: $\alpha = \frac{\pi}{4}$ et $\alpha + \pi = \frac{5\pi}{4}$. $S = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$.
4. $\cos x = 0$: $S = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$.

Synthèse

Solution 11.

1. $\tan x = -2 < 0$ et $\cos x > 0$: 4$^{\text{e}}$ quadrant ($-\frac{\pi}{2} < x < 0$ ou $\frac{3\pi}{2} < x < 2\pi$).
2. $1 + \tan^2 x = \frac{1}{\cos^2} x$ donne $\cos^2 x = \frac{1}{5}$, soit $\cos x = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ (positif). Et $\sin x = \tan x \cos x = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$.

Solution 12.

$\tan x + \frac{1}{\tan} x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$.