Corrigés — Calcul intégral

Chapitre 7

Solution 1.
1. $\left[x^3 - x^2 + x\right]_0^1 = 1 - 1 + 1 = 1$.
2. $[\sin x]_0^{\frac{\pi}{2}} = 1 - 0 = 1$.
3. $[\ln x]_1^e = 1 - 0 = 1$.
4. $\left[e^x\right]_0^1 = e - 1$.

Solution 2.
1. $u = x^2 + 1, u' = 2x$. Primitive : $\left(\frac{2}{3}\right)\left(x^2 + 1\right)^{\frac{3}{2}}$.
2. $u = \ln x, u' = \frac{1}{x}$. Primitive : $\frac{(\ln x)^2}{2}$.
3. $u = \cos x, u' = -\sin x$. $\int -u'u^2 \,\mathrm{d}x = -\frac{u^3}{3} = -\cos^3 \frac{x}{3}$.

Solution 3.
1. $u = x, v' = e^x$. $\left[xe^x\right]_0^1 - \int_0^1 e^x \,\mathrm{d}x = e - (e - 1) = 1$.
2. $u = \ln x, v' = 1$. $[x \ln x]_1^e - \int_1^e 1 \,\mathrm{d}x = e - (e - 1) = 1$.
3. $u = x, v' = \sin x$. $[-x \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \,\mathrm{d}x = 0 + 1 = 1$.

Solution 4.
$\left(\frac{1}{\pi}\right) \int_0^\pi \sin x \,\mathrm{d}x = \left(\frac{1}{\pi}\right)[-\cos x]_0^\pi = \left(\frac{1}{\pi}\right)(1 + 1) = \frac{2}{\pi}$.

Solution 5.
Aire $= \int_{-2}^2 \left(4 - x^2\right) \mathrm{d}x = \left[4x - \frac{x^3}{3}\right]_{-2}^2 = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) = 16 - \frac{16}{3} = \frac{32}{3}$.

Solution 6.
Sur $[0, 1]$, $x \geq x^2$. Aire $= \int_0^1 \left(x - x^2\right) \mathrm{d}x = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.

Solution 7.
$V = \pi \int_0^h \left(R\frac{x}{h}\right)^2 \mathrm{d}x = \left(\pi \frac{R^2}{h^2}\right) \left[\frac{x^3}{3}\right]_0^h = \left(\pi \frac{R^2}{h^2}\right) \times \frac{h^3}{3} = \frac{\pi R^2 h}{3}$. ✓

Solution 8.
$u = x^2 + 1, u' = 2x$. $\int_0^1 \frac{x}{x^2+1} \,\mathrm{d}x = \left(\frac{1}{2}\right) \left[\ln|x^2 + 1\right]\!\!]_0^1 = \left(\frac{1}{2}\right) \ln 2$.

Solution 9.
$I = \int_0^1 (\ln x)^2 \, dx$. IPP avec $u = (\ln x)^2$, $v' = 1$: $I = \left[x(\ln x)^2\right]_0^1 - \int_0^1 2 \ln x \, dx = 0 - 2[x \ln x - x]_0^1 = -2(0-1) = 2$ (on utilise $\lim_{x\to 0^+} x \ln x = 0$ et $x(\ln x)^2 \to 0$).

Solution 10.
Par le théorème fondamental : $F'(x) = \sqrt{1+x^2}$. $F''(x) = \frac{x}{\sqrt{1+x^2}}$.