Corrigés — Fonctions primitives

Chapitre 4

Solution 1.
1. $F(x) = x^3 - \frac{5x^2}{2} + x + c$.
2. $F(x) = \sin x - \cos x + c$.
3. $F(x) = -\frac{1}{x} + c$.
4. $F(x) = \left(\frac{2}{3}\right)x^{\frac{3}{2}} + c$.

Solution 2.
1. $u = x^2 + 1$, $u' = 2x$. $F = \frac{(x^2+1)^5}{5} + c$.
2. $f = \left(\frac{1}{2}\right)\frac{u'}{u}$ avec $u = x^2 + 1$. $F = \left(\frac{1}{2}\right)\ln(x^2 + 1) + c$.
3. $u = \sin x$, $u' = \cos x$. $F = \sin^3 \frac{x}{3} + c$.
4. $u = x^2 + 3x + 2$, $u' = 2x + 3$. $F = \ln|x^2 + 3x + 2| + c$.

Solution 3.
1. $F(x) = \frac{x^2}{2} - x + c$. $F(2) = 2 - 2 + c = c = 0$. $F(x) = \frac{x^2}{2} - x$.
2. $F(x) = \frac{\sin(2x)}{2} + c$. $F(0) = c = 1$. $F(x) = \frac{\sin(2x)}{2} + 1$.
3. $F(x) = x^3 - 2x + c$. $F(-1) = -1 + 2 + c = 5$, $c = 4$. $F(x) = x^3 - 2x + 4$.

Solution 4.
$f(x) = 1 + \frac{1}{x^2}$. Primitive : $F(x) = x - \frac{1}{x} + c$.

Solution 5.
$F(x) = 2\ln|x - 1| + c$. Sur $]1, +\infty[$: $F(x) = 2\ln(x - 1) + c$. $F(2) = 2\ln 1 + c = c = 0$. $F(x) = 2\ln(x - 1)$.

Solution 6.
1. **Admis** : $F(x) = \arctan(x) + c$.
2. $F(x) = \left(\frac{1}{3}\right)e^{3x+2} + c$ (car dérivée donne e^{3x+2}).

Solution 7.
$F' = f = G'$, donc $(G - F)' = 0$ sur l'intervalle. Une fonction de dérivée nulle sur un intervalle est constante. Donc $G - F = c$ constante.