

# Corrigés — Calcul intégral

## Chapitre 6

### Solution 1.

1.  $[x^3 - x^2 + x]_0^1 = 1 - 1 + 1 = 1$ .
2.  $[\sin x]_0^{\frac{\pi}{2}} = 1 - 0 = 1$ .
3.  $[\ln x]_1^e = 1 - 0 = 1$ .
4.  $[e^x]_0^1 = e - 1$ .

### Solution 2.

1.  $u = x^2 + 1, u' = 2x$ . Primitive :  $\left(\frac{2}{3}\right)(x^2 + 1)^{\frac{3}{2}}$ .
2.  $u = \ln x, u' = \frac{1}{x}$ . Primitive :  $\frac{(\ln x)^2}{2}$ .
3.  $u = \cos x, u' = -\sin x$ .  $\int -u' u^2 dx = -\frac{u^3}{3} = -\cos^3 \frac{x}{3}$ .

### Solution 3.

1.  $u = x, v' = e^x$ .  $[xe^x]_0^1 - \int_0^1 e^x dx = e - (e - 1) = 1$ .
2.  $u = \ln x, v' = 1$ .  $[x \ln x]_1^e - \int_1^e 1 dx = e - (e - 1) = 1$ .
3.  $u = x, v' = \sin x$ .  $[-x \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x dx = 0 + 1 = 1$ .

### Solution 4.

$$\left(\frac{1}{\pi}\right) \int_0^{\pi} \sin x dx = \left(\frac{1}{\pi}\right) [-\cos x]_0^{\pi} = \left(\frac{1}{\pi}\right)(1 + 1) = \frac{2}{\pi}.$$

### Solution 5.

$$\text{Aire} = \int_{-2}^2 (4 - x^2) dx = \left[4x - \frac{x^3}{3}\right]_{-2}^2 = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) = 16 - \frac{16}{3} = \frac{32}{3}.$$

### Solution 6.

$$\text{Sur } [0, 1], x \geq x^2. \text{ Aire} = \int_0^1 (x - x^2) dx = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

### Solution 7.

$$V = \pi \int_0^h \left(R \frac{x}{h}\right)^2 dx = \left(\pi \frac{R^2}{h^2}\right) \left[\frac{x^3}{3}\right]_0^h = \left(\pi \frac{R^2}{h^2}\right) \times \frac{h^3}{3} = \frac{\pi R^2 h}{3}. \checkmark$$

### Solution 8.

$$u = x^2 + 1, u' = 2x. \int_0^1 \frac{x}{x^2+1} dx = \left(\frac{1}{2}\right) [\ln|x^2 + 1|]_0^1 = \left(\frac{1}{2}\right) \ln 2.$$

**Solution 9.**

$I = \int_0^1 (\ln x)^2 dx$ . IPP avec  $u = (\ln x)^2$ ,  $v' = 1$  :  $I = [x(\ln x)^2]_0^1 - \int_0^1 2 \ln x dx = 0 - 2[x \ln x - x]_0^1 = -2(0 - 1) = 2$  (on utilise  $\lim_{x \rightarrow 0^+} x \ln x = 0$  et  $x(\ln x)^2 \rightarrow 0$ ).

**Solution 10.**

Par le théorème fondamental :  $F'(x) = \sqrt{1+x^2}$ .  $F''(x) = \frac{x}{\sqrt{1+x^2}}$ .