Corrigés — Équations différentielles

Chapitre 7

Solution 1.
1. $y(x) = Ce^{2x}$.
2. $y(x) = Ce^{-3x}$. $y(0) = C = 5$, donc $y(x) = 5e^{-3x}$.
3. $y(x) = Ce^{-x}$. $y(1) = \frac{C}{e} = e$, donc $C = e^2$, $y(x) = e^2 e^{-x} = e^{2-x}$.

Solution 2.
1. $y = Ce^x - 2$.
2. $y = Ce^{-2x} + 3$. $y(0) = C + 3 = 0$, $C = -3$. $y(x) = -3e^{-2x} + 3 = 3(1 - e^{-2x})$.
3. $y = Ce^{-3x} + 4$. $y(0) = C + 4 = 2$, $C = -2$. $y(x) = -2e^{-3x} + 4$.

Solution 3.
1. $y = A\cos x + B\sin x$.
2. $y(0) = A = 3, y'(0) = 3B = 0$, $B = 0$. $y = 3\cos(3x)$.
3. $y(0) = A = 0, y'(0) = 2B = 2$, $B = 1$. $y = \sin(2x)$.

Solution 4.
1. $N(t) = N_0 e^{-\lambda t}$.
2. $\frac{N_0}{2} = N_0 e^{-\lambda T} \Longrightarrow e^{-\lambda T} = \frac{1}{2} \Longrightarrow T = \frac{\ln 2}{\lambda}$.

Solution 5.
$P(t) = 1000e^{0\{,\}03t}$. $P(t) = 2000 \Longleftrightarrow e^{0\{,\}03t} = 2 \Longleftrightarrow t = \frac{\ln 2}{0}\{,\}03 \approx 23\{,\}1$ ans.

Solution 6.
$T(t) = Ce^{-0\{,\}05t} + 15$. $T(0) = C + 15 = 95$, $C = 80$. $T(t) = 80e^{-0\{,\}05t} + 15$. $T(20) = 80e^{-1} + 15 \approx 44\{,\}4$°C. Limite : 15°C (température ambiante).

Solution 7.
$(y_1 - y_2)' = y_{1'} - y_{2'} = (ay_1 + b) - (ay_2 + b) = a(y_1 - y_2)$. Donc résout $z' = az$.

Solution 8.
$y(x) = xe^x + Ce^x = (x + C)e^x$.

Solution 9.
$z = y'$ résout $z' + 2z = 0$, donc $z(x) = Ke^{-2x}$. Puis $y(x) = \int z = -\frac{K}{2}e^{-2x} + C = C_1e^{-2x} + C_2$.