Corrigés — Fonctions logarithmiques

Chapitre 4

Solution 1.
1. $\ln\left(\frac{8}{2}\right) = \ln 4 = 2\ln 2$.
2. 3.
3. $\frac{1}{2}$.
4. $2\ln 3 + 2\ln 2 - 2\ln 3 = 2\ln 2$.

Solution 2.
1. $x = e^3$.
2. $\ln((x-1)(x+1)) = 0 \Longrightarrow (x-1)(x+1) = 1 \Longrightarrow x^2 = 2 \Longrightarrow x = \sqrt{2}$ (condition : $x > 1$).
3. $X^2 - 3X + 2 = 0 \Longrightarrow X = 1$ ou $X = 2$. $x = e$ ou $x = e^2$.
4. $x \in]0, e[$.

Solution 3.
1. $f'(x) = \frac{2}{2x+1}$.
2. $f'(x) = 2x\ln x + x = x(2\ln x + 1)$.
3. $f'(x) = \frac{1-\ln x}{x^2}$.
4. $f'(x) = \frac{1}{x\ln x}$.

Solution 4.
1. $\frac{\ln x}{x} \to 0$ donc $\ln x - x = x\left(\frac{\ln x}{x} - 1\right) \to -\infty$.
2. 0 (croissance comparée).
3. 1 (limite remarquable).
4. $+\infty$.

Solution 5.
1. $f'(x) = \frac{1}{x} - 1 = \frac{1-x}{x}$. Zéro en $x = 1$. Max en $x = 1$, $f(1) = 0$.
2. $f(x) \leq f(1) = 0 \Longrightarrow \ln x - x + 1 \leq 0 \Longrightarrow \ln x \leq x - 1$.

Solution 6.
$D =]0, +\infty[$. $f'(x) = \frac{1-\ln x}{x^2}$. Zéro en $x = e$. Max $f(e) = \frac{1}{e}$. $\lim_{x\to 0^+} = -\infty$, $\lim_{x\to+\infty} = 0$. Asymptote horizontale $y = 0$ en $+\infty$.

Solution 7.
$\ln((x+1)^2) = \ln(x+3)$. $(x+1)^2 = x+3$, soit $x^2 + x - 2 = 0$, racines 1 et -2. Condition $x + 1 > 0, x + 3 > 0 : x > -1$. Garder $x = 1$.

Solution 8.
$\ln(x(x-2)) \geq \ln 3$ avec $x > 2$. $x^2 - 2x \geq 3$, soit $x^2 - 2x - 3 \geq 0$, racines $-1, 3$. Sur $x > 2 : x \geq 3$. $S = [3, +\infty[$.

Solution 9.
$f'(x) = 2x - \frac{2}{x} = \frac{2x^2-2}{x} = 2\frac{x^2-1}{x}$. Zéros en $x = 1$ (sur $]0, +\infty[$). Signe $-$ sur $]0, 1[$, $+$ sur $]1, +\infty[$. Minimum $f(1) = 1$.

Solution 10.
1. $\ln(1 + e^x) \geq \ln(e^x) = x$.
2. $\lim_{-\infty} = \ln(1) = 0 \quad \text{(asymptote} \quad \text{horiz} \quad y = 0). \quad f(x) - x = \ln(1 + e^x) - x = \ln\left(\frac{1+e^x}{e^x}\right) = \ln(e^{-x} + 1) \to 0$ quand $x \to +\infty$. Asymptote oblique $y = x$.