

Corrigés — Calcul trigonométrique

Chapitre 6

Solution 1.

- $\cos(15^\circ) = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$.
- $\sin(75^\circ) = \sin(45^\circ + 30^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$.
- $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$. $\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{3}\right) \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right) \sin\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{4}\right)(1 - \sqrt{3}) = \frac{\sqrt{2} - \sqrt{6}}{4}$.
- $105^\circ = 60^\circ + 45^\circ$. $\tan(105^\circ) = \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} = -\frac{(\sqrt{3} + 1)^2}{2} = -(2 + \sqrt{3})$.

Solution 2.

$$\begin{aligned} \cos(3a) &= \cos(2a) \cos a - \sin(2a) \sin a = (2 \cos^2 a - 1) \cos a - 2 \sin a \cos a \sin a = 2 \cos^3 a - \cos a - 2 \sin^2 a \cos a \\ &= 2 \cos^3 a - \cos a - 2(1 - \cos^2 a) \cos a = 2 \cos^3 a - \cos a - 2 \cos a + 2 \cos^3 a = 4 \cos^3 a - 3 \cos a. \end{aligned}$$

Solution 3.

$$\begin{aligned} \sin(2a) &= \sin(a + a) = \sin a \cos a + \cos a \sin a = 2 \sin a \cos a. & \cos(2a) &= \cos(a + a) = \cos a \cos a - \sin a \sin a = \cos^2 a - \sin^2 a. \end{aligned}$$

Utiliser $\cos^2 + \sin^2 = 1$ pour les deux autres formes.

Solution 4.

- $\cos^2 x = \frac{1 + \cos(2x)}{2}$.
- $\sin^2 x \cos^2 x = \left(\frac{\sin(2x)}{2}\right)^2 = \frac{\sin^2 2x}{4} = \frac{1 - \cos(4x)}{8}$.
- $\cos^3 x = \cos x \left(\frac{1 + \cos(2x)}{2}\right) = \frac{\cos x + \cos x \cos(2x)}{2}$. $\cos x \cos(2x) = \left(\frac{1}{2}\right)(\cos(x) + \cos(3x))$ (produit \rightarrow somme). Donc $\cos^3 x = \frac{3 \cos x + \cos(3x)}{4}$.

Solution 5.

- $\cos(3x) + \cos x = 2 \cos(2x) \cos x$.
- $\sin(5x) - \sin x = 2 \cos(3x) \sin(2x)$.
- $1 + \cos x = \cos 0 + \cos x = 2 \cos\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right) = 2 \cos^2\left(\frac{x}{2}\right)$.

Solution 6.

- $1 - 2 \sin^2 x + \sin x = 0 \Leftrightarrow 2 \sin^2 x - \sin x - 1 = 0$. $X = \sin x$: $\Delta = 9$, $X = 1$ ou $X = -\frac{1}{2}$.
 $\sin x = 1$: $x = \frac{\pi}{2} + 2k\pi$. $\sin x = -\frac{1}{2}$: $x = -\frac{\pi}{6} + 2k\pi$ ou $x = \frac{7\pi}{6} + 2k\pi$.
- $\cos(2x) = \frac{1}{2} \Leftrightarrow 2x = \pm \frac{\pi}{3} + 2k\pi \Leftrightarrow x = \pm \frac{\pi}{6} + k\pi$.
- $2 \cos\left(x + \frac{\pi}{6}\right) = 1$ (forme auxiliaire), soit $\cos\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$, donc $x + \frac{\pi}{6} = \pm \frac{\pi}{3} + 2k\pi$. D'où $x = \frac{\pi}{6} + 2k\pi$ ou $x = -\frac{\pi}{2} + 2k\pi$.

Solution 7.

1. $R = \sqrt{4 + 12} = 4$. $\cos \varphi = \frac{1}{2}$, $\sin \varphi = \frac{\sqrt{3}}{2}$, donc $\varphi = \frac{\pi}{3}$. $2 \cos x + 2\sqrt{3} \sin x = 4 \cos(x - \frac{\pi}{3})$.
Max = 4, min = -4.
2. $R = \sqrt{2}$, $\cos \varphi = \frac{1}{\sqrt{2}}$, $\sin \varphi = -\frac{1}{\sqrt{2}}$, $\varphi = -\frac{\pi}{4}$. $\cos x - \sin x = \sqrt{2} \cos(x + \frac{\pi}{4})$. Max $\sqrt{2}$, min $-\sqrt{2}$.

Solution 8.

$\cos(a - b) = \cos a \cos b + \sin a \sin b$; $\cos(a + b) = \cos a \cos b - \sin a \sin b$. Soustraire : $\cos(a - b) - \cos(a + b) = 2 \sin a \sin b$. Diviser par 2.

Solution 9.

$$\cos\left(\frac{2\pi}{5}\right) = 2 \cos^2\left(\frac{\pi}{5}\right) - 1 = 2 \times \left(\frac{1+\sqrt{5}}{4}\right)^2 - 1 = 2 \times \frac{6+2\sqrt{5}}{16} - 1 = \frac{3+\sqrt{5}}{4} - 1 = \frac{\sqrt{5}-1}{4}.$$

Solution 10.

Utiliser $\cos a \cos b = \left(\frac{1}{2}\right)(\cos(a - b) + \cos(a + b))$ pour chaque terme, puis sommer. Après calcul, on trouve $A = 0$ (les termes s'annulent par périodicité $\frac{2\pi}{3}$).