Corrigés — Dénombrement et probabilités

Chapitre 7

Solution 1.
1. $\binom{12}{4} = 495$.
2. $\binom{5}{2}\binom{7}{2} = 10 \times 21 = 210$.
3. $495 - \binom{7}{4} = 495 - 35 = 460$.

Solution 2.
Soit D : défectueux.
1. $P(D) = 0{,}4 \times 0{,}02 + 0{,}35 \times 0{,}03 + 0{,}25 \times 0{,}05 = 0{,}008 + 0{,}0105 + 0{,}0125 = 0{,}031$.
2. $P(F_3 \mid D) = \frac{0{,}25 \times 0{,}05}{0}{,}031 = 0{,}\frac{0125}{0}{,}031 \approx 0{,}403$.

Solution 3.
$P(\text{au moins un}) = 1 - 0{,}95^5 \approx 1 - 0{,}7738 \approx 0{,}2262$.

Solution 4.
$X \sim \mathcal{B}\left(10, \frac{1}{6}\right)$. $E(X) = \frac{10}{6} \approx 1{,}67$. $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - \left(\frac{5}{6}\right)^{10} - 10 \times \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{9}$. $\left(\frac{5}{6}\right)^{10} \approx 0{,}1615$, $\left(\frac{5}{6}\right)^{9} \approx 0{,}1938$. $P(X \geq 2) \approx 1 - 0{,}1615 - 10 \times 0{,}1667 \times 0{,}1938 \approx 1 - 0{,}1615 - 0{,}3230 \approx 0{,}5155$.

Solution 5.
$X \sim \mathcal{B}(20, 0{,}7)$. $E(X) = 14$, $V(X) = 20 \times 0{,}7 \times 0{,}3 = 4{,}2$. $P(X = 15) = \binom{20}{15}(0{,}7)^{15}(0{,}3)^5 = 15504 \times 0{,}004747 \times 0{,}00243 \approx 0{,}179$.