Corrigés — Calcul trigonométrique

Chapitre 6

Solution 1.

1. $\cos(15°) = \cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30° = \frac{\sqrt{6}+\sqrt{2}}{4}$.
2. $\sin(75°) = \sin(45° + 30°) = \frac{\sqrt{6}+\sqrt{2}}{4}$.
3. $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$. $\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{4}\right)\left(1 - \sqrt{3}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$.
4. $105° = 60° + 45°$. $\tan(105°) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} = \frac{\sqrt{3}+1}{1-\sqrt{3}} = -\frac{\left(\sqrt{3}+1\right)^2}{2} = -\left(2 + \sqrt{3}\right)$.

Solution 2.

$\cos(3a) = \cos(2a)\cos a - \sin(2a)\sin a = \left(2\cos^2 a - 1\right)\cos a - 2\sin a \cos a \sin a = 2\cos^3 a - \cos a - 2\sin^2 a \cos a = 2\cos^3 a - \cos a - 2\left(1 - \cos^2 a\right)\cos a = 2\cos^3 a - \cos a - 2\cos a + 2\cos^3 a = 4\cos^3 a - 3\cos a$.

Solution 3.

$\sin(2a) = \sin(a + a) = \sin a \cos a + \cos a \sin a = 2\sin a \cos a$. $\cos(2a) = \cos(a + a) = \cos a \cos a - \sin a \sin a = \cos^2 a - \sin^2 a$. Utiliser $\cos^2 + \sin^2 = 1$ pour les deux autres formes.

Solution 4.

1. $\cos^2 x = \frac{1+\cos(2x)}{2}$.
2. $\sin^2 x \cos^2 x = \left(\frac{\sin(2x)}{2}\right)^2 = \sin^2 \frac{2x}{4} = \frac{1-\cos(4x)}{8}$.
3. $\cos^3 x = \cos x\left(\frac{1+\cos(2x)}{2}\right) = \frac{\cos x + \cos x \cos(2x)}{2}$. $\cos x \cos(2x) = \left(\frac{1}{2}\right)(\cos(x) + \cos(3x))$ (produit $\to$ somme). Donc $\cos^3 x = \frac{3\cos x + \cos(3x)}{4}$.

Solution 5.

1. $\cos(3x) + \cos x = 2\cos(2x)\cos x$.
2. $\sin(5x) - \sin x = 2\cos(3x)\sin(2x)$.
3. $1 + \cos x = \cos 0 + \cos x = 2\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) = 2\cos^2\left(\frac{x}{2}\right)$.

Solution 6.

1. $1 - 2\sin^2 x + \sin x = 0 \iff 2\sin^2 x - \sin x - 1 = 0$. $X = \sin x$: $\Delta = 9$, $X = 1$ ou $X = -\frac{1}{2}$. $\sin x = 1 : x = \frac{\pi}{2} + 2k\pi$. $\sin x = -\frac{1}{2} : x = -\frac{\pi}{6} + 2k\pi$ ou $x = \frac{7\pi}{6} + 2k\pi$.
2. $\cos(2x) = \frac{1}{2} \iff 2x = \pm\frac{\pi}{3} + 2k\pi \iff x = \pm\frac{\pi}{6} + k\pi$.
3. $2\cos\left(x + \frac{\pi}{6}\right) = 1$ (forme auxiliaire), soit $\cos\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$, donc $x + \frac{\pi}{6} = \pm\frac{\pi}{3} + 2k\pi$. D'où $x = \frac{\pi}{6} + 2k\pi$ ou $x = -\frac{\pi}{2} + 2k\pi$.

Solution 7.

1. $R = \sqrt{4+12} = 4$. $\cos\varphi = \frac{1}{2}$, $\sin\varphi = \frac{\sqrt{3}}{2}$, donc $\varphi = \frac{\pi}{3}$. $2\cos x + 2\sqrt{3}\sin x = 4\cos\left(x - \frac{\pi}{3}\right)$. Max $= 4$, min $= -4$.
2. $R = \sqrt{2}$, $\cos\varphi = \frac{1}{\sqrt{2}}$, $\sin\varphi = -\frac{1}{\sqrt{2}}$, $\varphi = -\frac{\pi}{4}$. $\cos x - \sin x = \sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$. Max $\sqrt{2}$, min $-\sqrt{2}$.

Solution 8.
$\cos(a-b) = \cos a\cos b + \sin a\sin b$; $\cos(a+b) = \cos a\cos b - \sin a\sin b$. Soustraire : $\cos(a-b) - \cos(a+b) = 2\sin a\sin b$. Diviser par 2.

Solution 9.
$\cos\left(\frac{2\pi}{5}\right) = 2\cos^2\left(\frac{\pi}{5}\right) - 1 = 2 \times \left(\frac{1+\sqrt{5}}{4}\right)^2 - 1 = 2 \times \frac{6+2\sqrt{5}}{16} - 1 = \frac{3+\sqrt{5}}{4} - 1 = \frac{\sqrt{5}-1}{4}$.

Solution 10.
Utiliser $\cos a\cos b = \left(\frac{1}{2}\right)(\cos(a-b) + \cos(a+b))$ pour chaque terme, puis sommer. Après calcul, on trouve $A = 0$ (les termes s'annulent par périodicité $\frac{2\pi}{3}$).