Corrigés — Dénombrement et probabilités

Chapitre 12

Solution 1.
Arrangements de 5 parmi 26 : $A_{26}^5 = 26 \times 25 \times 24 \times 23 \times 22 = 7,893,600$.

Solution 2.
1. Arrangements de 3 parmi 10 : $A_{10}^3 = 720$.
2. Combinaisons : $\binom{10}{3} = 120$.

Solution 3.
1. $\binom{7}{3} = 35$; $\binom{10}{4} = 210$; $\binom{20}{2} = 190$.
2. $\binom{9}{2} + \binom{9}{3} = 36 + 84 = 120 = \binom{10}{3}$ ✓.

Solution 4.
$(x+2)^5 = x^5 + 5 \times 2x^4 + 10 \times 4x^3 + 10 \times 8x^2 + 5 \times 16x + 32 = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$.

Solution 5.
$|\Omega| = 36$.
1. Paires $(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)$: 6 cas. $P = \frac{6}{36} = \frac{1}{6}$.
2. Doubles : 6 cas. $P = \frac{1}{6}$.
3. Par linéarité : $E(\text{somme}) = 2 \times 3\{,\}5 = 7$. Pour P(pair) : $\frac{18}{36} = \frac{1}{2}$.

Solution 6.
1. $\frac{C(5,2)}{C(8,2)} = \frac{10}{28} = \frac{5}{14}$.
2. $\frac{C(5,1)\times C(3,1)}{C(8,2)} = \frac{15}{28}$.

Solution 7.
1. $P(A \mid B) = 0\{,\}\frac{2}{0}\{,\}5 = 0\{,\}4$.
2. $P(B \mid A) = 0\{,\}\frac{2}{0}\{,\}6 = \frac{1}{3}$.
3. $P(A) \times P(B) = 0\{,\}3 \neq 0\{,\}2 = P(A \cap B)$. **Non indépendants.**

Solution 8.
M = malade, T = test positif. $P(M) = 0\{,\}01$, $P(T \mid M) = 0\{,\}95$, $P\left(T \mid \overline{M}\right) = 0\{,\}10$ (complément de spécificité). $P(T) = 0\{,\}95 \times 0\{,\}01 + 0\{,\}10 \times 0\{,\}99 = 0\{,\}1085$. $P(M \mid T) = \frac{0\{,\}95\times 0\{,\}01}{0}\{,\}1085 \approx 0\{,\}087$, soit environ $8\{,\}7\%$.

Solution 9.

1. $0\{,\}1 + 0\{,\}3 + 0\{,\}4 + 0\{,\}2 = 1$. ✓
2. $E(X) = 0 + 0\{,\}3 + 0\{,\}8 + 0\{,\}6 = 1\{,\}7$. $E(X^2) = 0 + 0\{,\}3 + 1\{,\}6 + 1\{,\}8 = 3\{,\}7$. $V(X) = 3\{,\}7 - 2\{,\}89 = 0\{,\}81$. $\sigma = 0\{,\}9$.

Solution 10.

X suit une loi binomiale $B\big(3, \frac{1}{2}\big)$. $P(X = k) = \frac{\binom{3}{k}}{8}$.

- $P(X = 0) = \frac{1}{8}$; $P(X = 1) = \frac{3}{8}$; $P(X = 2) = \frac{3}{8}$; $P(X = 3) = \frac{1}{8}$.
- $E(X) = np = 1\{,\}5$; $V(X) = np(1 - p) = 0\{,\}75$.